**NON-ROUTINE PROBLEM-SOLVING SKILLS IN TRIGONOMETRIC IDENTITIES AMONG THIRD YEAR BACHELOR OF SECONDARY EDUCATION**

**MAJOR IN MATHEMATICS STUDENTS: BASIS FOR**

**A FRAMEWORK ON ALTERNATIVE THINKING**

**Ferrer, Norberto Jr M.1 and Caballes, Dennis G.2**

1National Teachers College, Manila, Philippines; 2Olivarez College Paranaque, Philippines

\*Correspondence Tel.: +639093340364

Email: 41800111@ntc.edu.ph

2d.caballes@olivarezcollege.edu.ph

***ABSTRACT:****This research investigates the creation and implementation of non-routine problem-solving skills in trigonometric identities among the students taking up Bachelor of Secondary Education, Third Year major in Mathematics at Pamantasan ng Cabuyao. It stresses the necessity to prepare future educators to think critically, creatively, and adaptively to solve complex mathematical problems. In the traditional teaching methods, the emphasis is given to the routine problem solving which hinders students’ potential to solve real life problems. This research also stresses on the need to develop a different thinking framework to solve problems.*

*Using a phenomenological approach, the study employed worded problem-solving tasks and semi-structured interviews to analyze students' strategies and cognitive processes. Findings revealed that students utilized systematic approaches, such as breaking problems into smaller steps, simplifying expressions, and applying fundamental trigonometric identities. Challenges included identifying appropriate identities and managing complex expressions, which students overcame through persistence, creativity, and reflective thinking.*

*The study proposes a framework for alternative thinking that emphasizes systematic approaches, adaptability, and critical analysis. This framework aims to enhance students' problem-solving skills and prepare them for modern educational and professional demands. The research contributes to mathematics education by offering insights into non-routine problem-solving and practical strategies for improving teaching and learning outcomes.*

**Keywords:***Non-routine problem-solving; Trigonometric identities; Systematic Approach; Critical thinking;Adaptive thinking; Creative problem-solving; Mathematics education*

**1. INTRODUCTION**

It is hence important in the field of studies to equip students with problem-solving skills as that which surpasses typical activities done in everyday learning. This is heft particularly when talking about the discourse on complex topics or subjects, for example, trigonometry where applied a real-world application usually requires deep understanding and the capability to solve mathematical conditions. This holds especially true for more complicated subjects like trigonometry, where real-life applications more often than not demand a deeper understanding and a capacity to deal with non-standard situations. In this regard, providing a schema upon which alternative thinking is going to be based is essential in enabling students to learn how to think critically and solve challenging problems in this case.

Identity problems in trigonometry are a vital portion of the Third Year Bachelor of Secondary Education Major in Mathematics.The students are able to identify the mathematical relationships and use them correctly in decision making. However, traditional pedagogical approaches often focus on standardized problem-solving approaches which may not enough to prepare students to meet the real-life challenges. Thus, the ability to solve non-routine problems is critical to the development of logical thinking, creative and critical reasoning, and decision-making abilities as required in real life situations [2, 3].

The Commission on Higher Education (CHED) focuses on the need to prepare students to meet personal and professional challenges. The solution to the given problem is not evident; thus, routine problem-solving processes are not applicable to it, in line with the objectives of the Commission on Higher Education (CHED) that encourage the development of critical thinking skills and creative solutions to problems. Solving a wide variety of mathematical challenges is part of a high level of proficiency in trigonometry, which helps students develop the skills they need to solve them and, therefore, advances their professional growth and viewpoints [6].

Many studies have been conducted on teaching pedagogies [7, 8, 9, 10, 11, 12], student preferences and readiness [13, 14], student motivation and attitude [15, 16, 17, 18], teachers skills, competencies, and challenges [19, 20, 21], assessment techniques and tools [22, 23, 24, 25] and other related factors [26, 27, 28, 29, 30, 31, 32] in order to enhance students learning outcome but little was done on investigating non-routine problem-solving skills specifically in trigonometric identities.

It is important to establish a framework for alternative thinking in order to enhance students' problem-solving skills in the field of trigonometry. This framework is intended to encourage students to use trigonometric concepts in non-standard ways, in ways that reflect a flexible manner of thinking, and from multiple perspectives. This approach encourages the development of creative and critical evaluation skills which are important in solving non-routine problems [2, 38].

The factors associated with teachers are important determinants of students’ performance in mathematics. Teacher attitudes, motivation and pedagogical proficiency are key determinants of student achievement, especially for understanding complex concepts. A framework based on non-routine problem-solving exercises along with real life examples has the potential to improve, to a great extent, students’ understanding of trigonometric concepts and create an environment of active learning [2, 36].

Outcome based education (OBE) is an innovative approach that Pamantasan ng Cabuyao (PnC) has adopted to improve the teaching and learning processes. Outcome Based Education is based on the student learning outcomes which are used as a reference for the development of curriculum, methods of teaching and assessment. This methodology is in line with the objective of enhancing students’ critical thinking and problem-solving skills in so far as problem solving in trigonometry is concerned [6].

However, challenges that are not identical to the examples provided in the trigonometric identities align more closely with the description of problems that require higher order thinking to solve because they cannot be solved through the application of surface level algorithms or concepts. This process helps to further develop students’ critical thinking skills, their flexibility, and their ability to work in unfamiliar environments. The ability to develop these skills is essential for future mathematics teachers to effectively convey information to their students [2, 39].

Research also highlights the effectiveness of using a more systematic approach to solving problems, such as Polya’s four step method, which are understand the problem, devise a plan, execute the plan, and analyze the solution. These strategies help students in the process of breaking down complex problems into their simpler components, which in turn helps them develop a more concept-based understanding of mathematics [2, 41].

Research shows that students have problems with atypical tasks, which is mainly due to them not using systematic and reflective thinking to solve the problems. As the tasks become more complex, the performance decreases which indicates the need for better pedagogical approaches and frameworks that promote the development of higher order cognitive processes [2, 38]. Thus, the present study aims to explore the relationship between students’ cognitive abilities and their performance on routine and non-routine problems within the context of instruction that incorporates expository, instructive, and interactive teachings. Through insights gained from the VLR model and SET, this paper will examine the challenges faced by students in solving problems, the impact of cognitive abilities on their performance, and the role of instruction in shaping this process. Furthermore, it will analyze the practices of teachers and educators that either facilitate or hinder the development of cognitive abilities necessary for solving routine and non-routine problems effectively.

It is important for effective solving of non-routine problems to only solve the problems within the context of this paper, the possession of prior knowledge and foundational skills is essential. Students who have a good grasp of mathematics are more likely to solve problems creatively as well as systematically. Therefore, it is important to build a good basis in algebra and precalculus to solve problems in trigonometry effectively [6, 38].

The ability to think creatively and to demonstrate adaptability appears to be critical for effective resolution of non-routine problems. Encouraging students to explore possible solutions and encouraging divergent thinking helps to develop students' flexibility and problem-solving skills. The qualities discussed above are critical in solving the complexity of trigonometric identities (4, 39).

Pattern recognition and visualization are very potent tools for improving the effectiveness of problem solving. The pedagogical strategies that include Worked examples, Group discussions with peers and Presentations of solutions are very effective in helping students to develop a good pattern recognition and use of concepts in new settings [39].

Persistence and adaptability are the most needed qualities in order to solve the not obviously stated problems. Resilient and adaptable students are more likely to face new challenges and find new ways of solving them. These qualities are very important in the attainment of success in mathematics education [40].

This paper highlights the importance of metacognition in solving non-routine problems as understanding one’s own thinking process is crucial. Making students reflect on their thinking process and the solutions they provide assists them to learn more and enhance their problem-solving skills. This methodology is consistent with the objectives of the present mathematics education [41].

The establishment of a framework for alternative thinking in trigonometry has the potential to effectively address the deficiencies present in contemporary teaching methodologies. Through the integration of systematic strategies, the cultivation of higher-order thinking skills, and the implementation of reflective practices, this framework has the potential to significantly enhance students' problem-solving capabilities and equip them for the challenges that lie ahead in their academic and professional journeys. This approach is consistent with the objectives set forth by CHED and contributes to the cultivation of proficient and innovative mathematics educators [2, 38, 39].

This study specifically sought to answer the following problems:

**Central Question:**

1. How do Third Year Bachelor of Secondary Education major in Mathematics students develop and apply non-routine problem-solving skills when faced with trigonometric identities?

**Corollary Questions:**

1. What specific strategies and cognitive processes do students employ to solve worded problems involving trigonometric identities using non-routine methods?
2. How do students' approaches to non-routine problem-solving in trigonometric identities reveal their understanding and application of critical thinking, creativity, and logical reasoning?
3. Based on the emerging themes, what framework in alternative learning in assessing solutions in trigonometric identities can be proposed?

**2. RESEARCH METHODOLOGY**

This study employed a phenomenological research design to explore the formation and application of non-routine problem-solving strategies in trigonometric identities among Third Year Bachelor of Secondary Education Major in Mathematics students at Pamantasan ng Cabuyao. The phenomenological approach was chosen because it focuses on the lived experiences of individuals, particularly how students navigate and make sense of complex mathematical situations. The purpose was to examine how students solved unconventional trigonometric problems that required critical thinking and problem-solving skills [2].

For data collection purpose, the study used problem solving tasks, and semi-structured interviews. This approach to the research gave a detailed view on how students solve problems and therefore was appropriate for use in this inquiry. The study was conducted to improve math instruction through the eyes of the students, with focus on the development of problem solving in atypical settings. The result provided significant knowledge to helped math teachers deal with problems of teaching mathematical concepts which involve problem solving, as well as creative and flexible thinking (3).

**2.1. Participants**

The research was conducted on third year students of Pamantasan ng Cabuyao, a state college, all taking Bachelor of Secondary Education with major in Mathematics. The entry criteria were set on pre-service teachers who are likely to teach mathematics in relatively near future. The students were concerned with improving the likelihood of handling difficult mathematical concepts such as trigonometric identities, the focus of this study.

For the current study, purposive sampling was used to select students who had taken trigonometry and were willing to solve problem solving tasks and follow up interviews. Then a simple random sampling was used to select 9 participants from the large sample. These participants were assigned into three groups based on their achievement in school: High performing students (Grades 90–100), Average performing students (Grades 80–89) and Low performing students (Grades <80).This grouping enabled us to get a good variation in problem solving capacity and to get a better picture of how students with different potential solved non routine problems [34].

This division into populations and the sampling of the approach enabled a proper assessment of how students in Third Year Bachelor of Secondary Education in Mathematics acquired and applied specific problem-solving skills in relation to trigonometric identities. The finding from this sample has implications for math teachers on how they can enhance their students’ readiness to solve math problems and explain them to their students in the classroom.

**2.2. Research Instrument**

The study employed two main data collection tools; these were written problem solving tasks and semi- structured interviews. The problem-solving tasks were developed to determine students' capacity to use trigonometric identities in non-routine forms problems. Some of the tasks included were solving problems that involved basic identities, angle sum and difference identities and other more complex trigonometric concepts. They were written in natural language and thus called for students to use their thinking skills and to go beyond the normal problem-solving approaches [5].

Semi-structured interviews were conducted after the problem-solving tasks to gather more detailed information concerning the thinking and planning of the participants. The interviews were conducted to help the researcher identify the strategy used by the students in solving the problems, the difficulties they encountered and the strategies they used to solve those difficulties. This qualitative data provided a more detailed description of the participants' experiences and helped in the development of a cognitive framework that can be used to improve students' problem-solving skills in trigonometry [4].

**2.3. Data Gathering Procedure**

In the first stage, in the form of a written problem-solving activity and structured interviews, third year Mathematics students of Pamantasan ng Cabuyao were asked how they solve trigonometry problems. With the help of the students’ written responses, they were able to identify what they do in solving trigonometry problems. The research method involves students independently solving non-routine word problems which involve the use of trigonometric identities like angle sum identities, angle double identities and product to sum identities. It is evident from their written responses that they critically and creatively applied mathematical concepts in new contexts.

In the second stage, formal interviews were conducted to explore in more detail the thinking processes, plans, difficulties and facilities that students employ during problem solving. These interviews were conducted in a calm environment and were additional to the written responses. Thus, the two methods together gave a full view of the process and the manner in which students develop and employ nonroutine problem solving skills in trigonometry.

**2.4. Thematic Analysis Process**

The study used a six steps thematic analysis technique which was in sync with the current studies in the area of mathematics education [4, 5, 41]. The researcher started with the data, which involved transcribing and then reading through the semi structured interviews numerous times in an effort to identify patterns and insights [4]. Initial codes were created by categorizing data snippets into themes, such as systematic problem solving', 'creative strategies' and 'persistence' and achieving accurate manual coding [5]. Then, the codes were aggregated into wider levels, e.g. systematic problem‐ solving approaches, in order to effectively capture the data [41].

After coding the themes, they were reviewed for coherence and relevance by going back to the transcripts and resolving any discrepancies through refinement where ever necessary [4]. Then each theme was defined and named to relate to the central ideas, for example, in the context of systematic problem solving which was described as the ability to break down tasks into steps and develop plans [5]. Last, the themes were incorporated into the results and discussion sections and participant quotes were used to support the findings and address the research questions [41]. This is a structured way of processing the data and thus ensures that the data is thoroughly and accurately analyzed.

**3. RESULT AND DISCUSSION**

**3.1. Systematic Problem-Solving Approach**

Students used a systematic approach to solve trigonometric identity problems, breaking them down into smaller, more manageable parts and applying basic trigonometric identities to find solutions. Their responses emphasized how important it is to take that first step in problem-solving before moving on to find a solution. Student 1 mentioned,

*“I always try to express the terms in sine and cosine form, as it makes the problem easier to handle.”*

In a similar vein, Student 6 expressed,

*“This method allows me to steer clear of errors by breaking down the problem into smaller parts to make it easier to understand.”*

This approach ensured that the themes were distinct and contributed to the clarification of the data. For instance, systematic problem-solving involves dividing tasks into several steps and coming up with reasonable plans to solve problems [5].

Students obviously used a systematic way of verifying their solutions. Student 7 said,

*“I check each step to see if my transformations are correct and if the left-hand side is equal to the right-hand side.”*

This is a style of thinking that is quite rational and systematic, and is used mainly for non-routine problems and for verifying the solutions. The students were able to recognize patterns and solve complex mathematical problems by utilizing logical reasoning due to the structured problem-solving approach taught to them. This method helped students ‘uncomplicate’ the problems and helped them feel good about the solutions they were coming up with [5].

Furthermore, students showed the potential to change their problem-solving strategy depending on the parts of the challenge. Student 9 said,

“*I first check what the problem looks like to see if it has square or fractional-like characteristics and then look for the trigonometric identity that fits it.”*

Thus, the structured framework was flexible enough to assist students in solving problems in a step-by-step process and make fewer mistakes. The focus on the requirement for a systematic approach to problem-solving is a direct reflection of the importance of foundational knowledge in trigonometry since the students had to use their knowledge of basic identities to solve the solutions. Structured methods are very useful for students who are facing new tasks because they provide students with a simple way of viewing tasks [41].

One of the most crucial characteristics of the strategies used by students in solving problems was the way of addressing complicated problems. They gave a way of breaking down problems to the basic level, and every step was taken carefully. This method not only assisted in reducing some errors to some extent but also helped in developing students' confidence and their problem-solving skills. Systematic problem-solving is a good cognitive strategy that helps students solve mathematical problems [2, 5, 41].

**3.2. Role of Prior Knowledge**

Prior knowledge was seen as one of the major determinants of the strategies that students use in solving problems. A large number of students used their knowledge of basic trigonometric identities and algebraic methods in order to solve the difficult problems. Student 4 said,

*“I went back to the basics; I looked at the basic trigonometric identities that I learnt in my first year of university and those include the Pythagorean identities and the Reciprocal identities, and I used them to solve the problems.”*

In the same vein, Student 9 said,

*“First I solved the problem and then compared it with a square or fraction to see which of the trigonometric identities could be similar to it.”*

The result reveals that prior knowledge was utilized as a framework of reference in solving math problems, so that students could relate new math problems to previous knowledge. This emphasis on prior knowledge is critical for solving non-routine problems and facing new problems with courage [4].

Furthermore, when solving a particular kind of problem, students referred to their notes or previous knowledge. Student 2 said,

*“When you meet a new identity, I try to explain it in terms of other similar identities or note down the answer.”*

This evidence of active retrieval and reference means that their comprehension of it is key to their application to non-routine problems. Students who practice reviewing and applying previous knowledge are likely to be successful in complicated mathematical tasks [35].

The students’ prior knowledge assisted them in making associations and understanding the relationships between the two subjects. Student 6 said,

*“This experience has underlined the role of critical thinking and the pattern recognition of items we had not even considered.”*

The ability to identify patterns is also important for the development of mathematical reasoning so that students are to be able to know what approach they should use to solve a given problem. This is because pattern recognition is critical for solving non-routine problems since it helps students to develop a systematic way of thinking about the possible solutions in the process of looking for logical solutions [5].

It was found that prior knowledge is a significant predictor of the strategies that students use in solving problems. It assisted them in the formulation and expression of their ideas, such that they approached non-routine problems with courage and creativity. When prior knowledge was used, students were able to identify patterns, make comparisons with other situations, and, therefore, provide possible solutions to the problems they faced.

**3.3. Persistence and Overcoming Challenges**

Persistence is a critical factor in solving non-routine problems, as it enables students to overcome challenges and maintain focus when faced with unfamiliar or complex tasks. Students reported feeling overwhelmed at times but emphasized the importance of perseverance. For example, Student 1 said,

*"When I get stuck, I step away and then try to solve the problem in small parts."*

This approach allowed students to simplify the problem and gain a clearer understanding of the task. Similarly, Student 9 explained,

*"I explain the statement and try to make it simpler, based on the language used. I then try to find what I can relate to from our daily lives."*

Persistence, combined with a step-by-step approach, helped student’s complete problem sets and overcome difficulties [2].

Students also found ways to manage frustration and maintain concentration. For instance, Student 6 shared, "I take regular breaks to reduce feelings of anxiety and overwhelm and to look at the problem from a different angle." This ability to manage stress and maintain focus reflects the development of a growth mindset, which is essential for solving non-routine problems. Persistence and resilience are key qualities that enable students to learn from their mistakes and refine their problem-solving strategies [40].

Additionally, persistence allowed students to explore alternative methods when their initial strategies failed. For example, Student 8 said, "When I forgot the double angle identity, I used sum and difference identities to solve the problem." This flexibility in problem-solving demonstrates students' ability to adapt and try different approaches. A positive educational environment that encourages students to make mistakes and learn from them fosters creativity and adaptability, which are essential for solving non-routine problems [5, 40].

Through persistence, students were able to experiment with different approaches, learn from their experiences, and build confidence in their problem-solving abilities. This mindset encouraged resilience and adaptability, which are crucial for tackling complex non-routine problems. By persisting through challenges, students developed their mathematical reasoning and problem-solving skills, preparing them for future learning and problem-solving scenarios [38].

**3.4 Creative and Adaptive Problem-Solving Strategies**

In solving non-routine problems, students showed originality and flexibility in their problem-solving heuristics, especially when the problems were unusual or complex in nature. A number of students explained why innovation is important for creating new identities or using unorthodox approaches. Student 5 said,

*“I had to use my intelligence to explain and make fractions of fractions easier to understand, which meant using other mathematical ideas.”*

Student 8 explained how they solved a complex problem, saying,

*“I broke the problem down into its components and then proceeded to use the tangent and cotangent identities freely to solve the problem.”*

The responses indicate students’ capacity to alter their problem-solving strategies depending on the kind of problem they are faced with [4].

Students’ reaction to the situation was to see what other ways they could use to solve the problem when the first could not work. Student 2 said,

*"When I had forgotten the double angle identity, I used sum and difference identities to solve the problem."*

This flexibility in the approach to the problems is evidence that students are able to change their typical approaches and look for other ways to solve the problems. Adaptability is one of the elements of non-routine problem-solving, which is critical for assisting students in solving problems and producing new strategies [5].

Furthermore, students also used algebraic manipulation and visualization techniques effectively to solve the problems. For example, Student 7 said,

*“I just added up fractions and moved some terms to the other side of the equal sign so that the equation became easier to solve.”*

This approach shows how students are using their basic algebra and trigonometry knowledge to simplify problems and look for patterns. Effective creative problem-solving is usually a combinative approach, and the problems are considered from different angles to help students grasp the mathematical ideas on a deeper level [41].

**3.5 Growth in Critical Thinking and Mathematical Reasoning**

The performance of the students on items that assessed them on routine and non-routine problems showed positive developments in their critical thinking and mathematical reasoning. A large number of students were found to be competent in problem analysis, pattern recognition, and methodical reasoning throughout each step. Student 7 said,

*“I look at the given equation, understand some basic identities, for example, sine squared theta + cosine squared theta = 1, and then try to solve it by breaking down the equation into parts to solve it systematically.”*

This is a systematic approach to the problem, and all the transformations are logical and geared towards solving the problem. Student 3 said:

*“I try to rewrite all the concepts in terms of sine and cosine as this is where I am most comfortable, and it helps to reason through the problem more easily.”*

Students’ interaction with non-routine problems enhances their reflective thinking and logical reasoning, which in turn enhances their confidence and accuracy in problem-solving [36].

The students said that solving non-routine problems assisted them in the ability to recognize patterns and relate different mathematical concepts. Student 6 said that the experience helped them to improve their critical thinking skills and to see patterns that they have never seen before. Hence, the development of critical thinking and reasoning was not only applied beyond trigonometric identities but also enhanced the students’ mathematical competence in general. Critical thinking and pattern recognition are vital to solving non-routine problems because they help students approach problems systematically and come up with potentially good solutions [5].

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**Figure 1. Framework for Enhancing Non-Routine Problem-Solving Skills in Trigonometry**

1. **Step-by-Step Analytical Approach**is important in solving complex trigonometric problems. This method involves breaking down problems into smaller, manageable parts and then systematically solving them by applying known trigonometric identities like the Pythagorean Identity or Reciprocal Identity. Students are encouraged to simplify expressions into either sine or cosine form as this makes the problem-solving process more intuitive. Guided practice and instructor modeling are crucial in assisting students in developing this systematic approach. Students learn by doing and confirming each transformation solved while solving problems, which helps them gain confidence and accuracy in their solutions. This approach has been found to improve students’ mathematical reasoning and problem-solving skills. [5].
2. **Encouraging Creativity and Adaptability**are essential in problem solving because they involve types of problems that are not routine and thus require students to use ideas that are not conventional. Students are also expected to try other approaches e.g. using conjugates, factoring or even the identities combined to solve the problem. For example, where a standard identity cannot be used, students may need to transform the expression or try several solutions. This flexibility helps to develop critical thinking and problem-solving skills. Students’ capacity to solve novel problems is also improved through creative exercises and problem variations to help them solve problems with creativity and boldness [5,4].
3. **Building on Prior Knowledge**is important in solving non-routine problems since the student is able to relate new concepts with previous knowledge. In this way, students are able to make sense of the new concepts being taught since they are able to link them with their prior knowledge of fundamental trigonometric identities such as the Pythagorean and Quotient identities. Some strategies include active recall exercises and concept mapping which help the students to build the connection between different identities and use them in new situations. In conclusion, those students who use prior knowledge correctly are likely to solve problems correctly and with less anxiety and more creativity [35].
4. **Persistence and Coping with Challenges**is a key factor in solving non-routine problems and overcoming the challenges that come with it. Students are told to step back from the problem when they get stuck, make the problem simpler and try something different. Students’ stress is managed through techniques like breaking away or approaching problems with a new outlook. Thus, a growth mindset helps students to develop persistence and resilience, so that they can look at any difficulty as a learning experience. Peer collaboration and mentorship also help students in the persistence of persistence, since they can share ideas and encourage each other to solve the problems [40].
5. **Developing Critical Thinking and Mathematical Reasoning**are essential for solving trigonometric identity problems. Students are taught to decompose problems into logical steps, recognize patterns, and apply logical reasoning to arrive at solutions. Exercises that require students to analyze and critique problem-solving steps help them develop a deeper understanding of the underlying concepts. Non-routine problem-solving promotes metacognition and innovation, enabling students to explore multiple perspectives and refine their reasoning skills. By fostering critical thinking, students not only improve their problem-solving abilities but also prepare for future challenges in mathematics and beyond [38].

**4. CONCLUSIONS**

The study starts with the problem and the conclusions drawn from the result for the study.

1. The results indicate that students use both a range of heuristics as well as cognitive creativity to solve the non-routine trigonometric identity problems. First, they start by solving the problem and simplifying it as a first step and sometimes they try to change the given terms into sine and cosine for the expressions to be solved to be easier. Thereafter, they use basic trigonometric identities such as the Pythagorean, Reciprocal and Quotient Identities in order to solve the problem further. But when the problem is hard or when it concerns new identities, the student becomes more inventive and may try out different approaches like multiplying by conjugates, factoring or deriving new identities from the given ones. This approach proves their flexibility as they are able to come up with new ideas when solving the problems. Some of the cognitive processes involved are analysis, problem solving, and critical thinking which are necessary in solving problems that are not standard. In all, students apply both habitual problem-solving techniques and innovative thinking to deal with the challenges of solving non-routine problems in trigonometry.
2. Students find it difficult to deal with non-routine trigonometric identity problems and at the same time find it as a satisfying exercise. The majority of students complained of being overwhelmed by the problems especially by the ones that were new to them or the ones that needed a lot of algebraic work. Nevertheless, students stated that isolating the problem in various ways made it easier for them to solve it. They also stressed on the need for patience and flexibility. To solve the difficulties, students usually stepped away from the problem, go back to their notes, or try another strategy which they thought could work. This is because the spirit of perseverance made the pupils stick with the problem solving until they were able to get the right solutions to the problems. In terms of growth, pupils were able to enhance their critical thinking and mathematics skills through the process of solving non-routine questions. Some students said that they enhance their confidence in problem solving and learned how to solve problems more creatively but logically. All in all, the students were able to enhance their understanding of trigonometric identities and their problem-solving ability
3. From the students’ experience with non-routine trigonometric problems, certain major themes were identified. First, there was a lot of focus on process – students used simplification of expressions and application of trigonometric identities to solve for simpler problems. This theme captures the need for a logical and sequential way of thinking through solving the problems that are not standard. Another important finding is the students’ problem-solving skills and their ability to be innovative and flexible in solving the problems. Some of the student’s strategies include the use of factors, multiplying by conjugates or coming up with new identities highlighting the flexibility of the students. Also, prior knowledge is greatly involved in their capacity to solve difficult problems. It was common for students to use knowledge of the basic trigonometric identities to help with harder problems; students used what they knew to help with parts that they did not understand when solving a problem. Persistence and coping with challenges also appeared as important factors. However, students showed perseverance by solving problems, reviewing concepts and solving the same problem using different approaches. Finally, the students’ experience revealed improvement in the critical thinking and mathematical skills. It can be noted that during the work with the non-routine problems, students enhanced their analytical skills, learned how to recognize patterns and, thus, improve their mathematical competencies. These themes capture the complexity of the problem-solving process in trigonometry and the processes through which students built up the skills to deal with problems that were not routine.
4. From the experiences and themes highlighted, a framework for the improvement of non-routine problem-solving skills in trigonometry can be suggested. This framework should consist of five components. First, a systematic approach should be promoted where students first identify and solve for the parts of the problem, simplify expressions and apply identities to the problem in a step-by-step manner. Second, the framework should also help to develop methods, creativity such and as flexibility, factoring so and that using students’ conjugates would when learn solving not the only problems how that to do use not the fit most the commonly standard used patterns. Third, it is crucial to build on knowledge since the students’ knowledge of the basic trigonometric identities acts as a backbone while solving the problems. This should be enhanced through the practice of going back through the fundamentals and linking them to the new content. Fourth, it is also important to focus on persistence and coping with challenges. Students should be taught to solve problems by subdividing them into several parts, to pause and continue working when one is fatigued and to look for other strategies when stuck. Finally, the framework should also be to develop critical thinking and mathematical reasoning where practice of logical reasoning, recognition of patterns, and assessment of multiple answers are enhanced. With these components, the framework will assist the students to enhance their non-routine problem-solving skills in trigonometry and hence prepare them to deal with other mathematical problems which may be presented to them.

**5. RECOMMENDATION**

The recommendation drawn from this study were based from the significance of the study along with the result of this study.

1. **Curriculum Planners:** it is recommended that curriculum planners incorporate more focus on **problem-solving strategies** and **critical thinking** within the trigonometry curriculum. The study revealed that students rely heavily on fundamental identities like the Pythagorean and Reciprocal Identities but often struggle when faced with non-routine problems that require creative and adaptive thinking. Curriculum planners should design learning competencies that emphasize not only the memorization of trigonometric identities but also the **application of these identities in varied, non-routine contexts**. This would help students to develop creativity, flexibility and critical thinking skills and thus to help them to develop to be able to solve complicated real-life problems. Furthermore, interdisciplinary approaches that link trigonometry to other areas of mathematics and real-world applications should also be introduced to make sure that students can use their knowledge in a more extensive way.
2. **Administrators:** For math teachers, it is recommended that administrators fund professional development initiatives that increase the likelihood that they can teach non-routine problem-solving. Students benefit from solving problems following the systematic, creative and flexible manner described above. Therefore, administrators should support and help educators with the resources they need to incorporate the latest teaching methods for both methodical and creative approaches in the classroom. The workshops or seminars that focus on critical thinking, problem solving strategies, and how to apply trigonometry in real life settings can also be organized. Administrators should also make sure that students' critical thinking and problem-solving skills are continuously evaluated in order to monitor their development and modify their teaching strategies as necessary.
3. **Mathematics Teachers:** Teachers of mathematics should place an emphasis on active learning tactics that allow students to participate in problem-solving exercises that involve both routine and non-routine trigonometric issues. According to the findings of the study, students frequently rely on conventional approaches, but they have difficulties when they are required to think creatively or adaptably instead. Problems that require factoring, the use of conjugates, or the derivation of new identities are examples of the kinds of exercises that teachers should add into their lessons in order to challenge students to think beyond the identities they have remembered. Teachers should also encourage students to work together on problems and to solve problems in groups, since this can help students develop their creative and critical thinking skills. It would also be beneficial to students' understanding if they participated in regular reflection exercises, in which they explained the procedures by which they solved problems. As a final recommendation, it is suggested that professors provide students with a greater number of real-world applications of trigonometry in order to demonstrate to them how these skills are utilized outside of the classroom.
4. **Students:** When it comes to students, it is strongly suggested that they actively participate in exploratory problem-solving activities in order to improve their cognitive abilities and creative abilities. According to the findings of the study, students have a tendency to rely on familiar identities and procedures when confronted with non-routine challenges, which can be overwhelming. Students should constantly practice solving non-routine problems, gradually progressing from simpler to more difficult assignments, in order to strengthen their problem-solving skills. In order for students to develop the confidence necessary to apply the fundamental trigonometric principles in a variety of contexts, it is essential for them to periodically examine these concepts themselves. They will be able to cultivate a mindset that is more adaptable if they make it a practice to break down difficulties into smaller, more manageable sections and experiment with different techniques. Students should engage in collaborative learning by participating in group discussions or study groups in order to share and explore different approaches to how to solve problems. For the purpose of improving tenacity in problem-solving, it is vital to cultivate a growth mindset, which is characterized by the perception of challenges as chances for improvement.
5. **Future Researchers:** This study establishes a basis for future researchers to investigate alternative problem-solving frameworks in trigonometry. Future research may examine the effects of particular teaching methods that combine routine and non-routine problem-solving techniques, and assess whether these methods enhance students' overall problem-solving skills. Future research may investigate the impact of technology-based tools and mathematical software on students' problem-solving abilities in trigonometry. Researchers may investigate longitudinal studies that monitor the development of critical thinking and creativity in students as they engage with trigonometric problems over time, yielding insights into the evolution of these skills through sustained practice. Exploring the role of interdisciplinary learning, wherein students relate trigonometric concepts to fields such as physics, engineering, or real-world data, may provide valuable insights into improving student engagement and problem-solving skills.

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**REFERENCES**

[1] Ahmad Shahril, N. S. S., Ahmad, S., & Arshat, Z. (2021). Parenting Styles, School Connectedness and Mental Health Among Adolescents in Selangor, Malaysia. International Journal of Academic Research in Business and Social Sciences, 11(6). DOI: 10.6007/ijarbss/v11-i6/10217

[2] Akyüz, G. (2020). Non-routine problem-solving performances of mathematics teacher candidates. Educational Research and Reviews, 15(5), 217-225. <https://dx.doi.org/10.5897/err2020.3907>

[3] Jatiningtyas, P. D., Kartono, &Mindyarto, B. (2022). Test Instruments to Measure Non-Routine Mathematics Problem Solving Ability Grade IV Elementary School Students. Journal of Educational Research and Evaluation, 6(3), 205-213. <https://dx.doi.org/10.23887/jere.v6i3.48656>

[4] Murphy, P. E. L., Evans, T., Klymchuk, S., Novak, J., Stephens, J., & Thomas, M. (2020). University STEM students' perceptions of creativity in non-routine problem-solving. ANZIAM Journal, 61, C345-C364. <https://dx.doi.org/10.21914/anziamj.v61i0.15052>

[5] Nguyen, H. A., Guo, Y., Stamper, J. C., & McLaren, B. (2020). Improving Students’ Problem-Solving Flexibility in Non-routine Mathematics. In G. Nejat, B. K. Smith, F. A. Amaya, & O. Khatib (Eds.), Robotics: Science and Systems XVI (pp. 1045-1052). Springer. <https://dx.doi.org/10.1007/978-3-030-52240-7_74>

[6] Padernal, R. E., & Diego, C. V. (2020). Academic Performance of Senior High School Students in Pre-Calculus. Philippine Social Science Journal, 3(2), 69-70. <https://doi.org/10.52006/main.v3i2.185>

[7] Gayeta, N. E. &Caballes, D. G. (2017). Measuring conceptual change on stoichiometry using mental models and ill-structured problems in a flipped classroom environment. *Asia Pacific Journal of Multidisciplinary Research*, *5*(2), 104-113.

[8] Guiao, C. D., & Caballes, D. G (2023). Teachers’ perception on integrating historical vignettes in teaching science concepts. *International Journal of Innovative Science and Research Technology, 8*(1), 1985-1990.

[9] Narca, M. L, & Caballes, D. G. (2021). Exploring students’ mental constructs on evolution towards proposed pedagogical interventions. *International Journal of Science and Research, 10*(8), 857-862.

[10] Caballes, D. G., Panol, R. F., Vasquez, A. G., & Valdez, M. R. (2022). Offline modular learning in a public school system: its perceived effects on school operations. *International Journal of Research in Engineering and Science, 10*(3), 21-26.

[11] Ucang, J. & Tan, D.A. (2013). Students’ Beliefs and Mathematics Performance in a Process-Oriented Guided-Inquiry Learning (POGIL) Environment. *CMU Journal of Science.* 17 (2013), 141-157.

[12] Florungco, J. K. E. &Caballes, D. G. (2021). A narrative study of science teaching methods and techniques in the new normal*. International Journal of Asian Education, 2*(3), 296-303.

[13] Ong, A. K. S., Prasetyo, Y. T., Chuenyindeedobla, T., Young, M. N., Doma, B. T., Caballes, D. G., Centeno, R. S., Morfe, A. S., & Bautista, C. S. (2022). Preference analysis on the online learning attributes among senior high school students during the COVID-19 pandemic: a conjoint analysis approach. *Evaluation and Program Planning, 102100*.

[14] Caballes, D. G., &Tabang, M. P. (2022). Grade 10 students' online learning readiness and e-learning engagement in a science high school during a pandemic. *Journal of Humanities and Education Development, 4*(3), 237-241.

[15] Narca, M. L., & Caballes, D. G. (2021). Learning motivation: strategies to increase students’ engagement in online learning at San Sebastian College-Recoletos, Manila. *International Journal of Asian Education, 2*(4), 573-580.

[16] Aguanta, E. & Tan, D.A. (2018). Effects of Dyad Cooperative Learning Strategy on Mathematics Performance and Attitude of Students Towards Mathematics, *International Journal of English and Education*, 7(3), 303-313.

[17] Ciubal-Fulgencio, N., & Tan, D. (2018). Effects of mathematics communication strategies on attitude and performance of grade 8 students, *Asian Academic Research Journal of Multi-disciplinary*, Volume 5, Issue 2, 44-53, February 2018.

[18] Cordova, C., & Tan, DA. (2018). Mathematics Proficiency, Attitude and Performance of Grade 9 Students in Private High School in Bukidnon, Philippines‖. *Asian Academic Research Journal of Social Sciences and Humanities*, vol. 5, issue 2, pp. 103-116, February 2018.

[19] Doblada, J. C. L. &Caballes, D. G., (2021). Relationship of teachers’ technology skills and selected profile: basis for redesigning training for online distance learning modality. *Instabright International Journal of Multidisciplinary Research, 3*(1), 17-22.

[20] Caballes, D. G., Panol, R. F., Vasquez, A. G., & Valdez, M. R. (2021). Competency level of science teachers in teaching evolution: basis for training design. *Global Journal of Advanced Research, 8*(8), 235-243.

[21] Herrera, M. B., &Caballes, D. G. (2022). Challenges of teachers amidst sustained global health crisis. Journal of Humanities and Education Development 4 (3), 142-149*, 4*(3), 142-149.

[22] Cordova, C., Pagtulon-an, EA., & Tan, DA. (2018). No Assignment Policy: A Boon or A Bane?‖. *International Journal of English and Education*, 8(1), 144-160, January 2019.

[23] Cordova C., Tan D. and Ucang J. (2018). Take Home Assignment and Performance of Grade 11 Students. *International Journal of Scientific and Technology Researches,* 7(12), 57-61, December 2018*.*

[24] Pagtulon-an, E. & Tan D. (2018). Students’ Mathematics Performance and Self-efficacy Beliefs in a Rich Assessment Tasks Environment. *Asian Academic Research Journal of Multidisciplinary*. 5(2), 54-64.

[25] Tan, D.A., Cordova, C.C., Saligumba, I.P.B., Segumpan, L.L.B. (2019). Development of Valid and Reliable Teacher-made Tests for Grade 10 Mathematics. *International Journal of English and Education*, 8(1), January 2019, 62-83.

[26] Jackaria, P. M., &Caballes, D. G. (2022). Equipping teachers to adapt: a look into teachers’ professional development experiences in times of COVID-19 pandemic. *Journal of Humanities and Education Development, 4*(4), 18-22.

[27] Panol, R. F., Vasquez, A. G., Valdez, M. R., &Caballes, D. G., (2021). Parental involvement on students’ completion of learning tasks in science. *International Journal of Scientific Research in Multidisciplinary Studies, 7*(5), 1-7.

[28] Tan, D. A., &Balasico, C. L. (2018). Students’ Academic Performance, Aptitude and Occupational Interest in the National Career Assessment Examination. *PUPIL: International Journal of Teaching, Education and Learning*, 2(3), 01-21.

[29] Tan, D.A. (2018). Mathematical Problem Solving Heuristics and Solution Strategies of Senior High School Students, *International Journal of English and Education*, 7(3), July 2018, 1-17.

[30] Duque, C. & Tan, D. (2018). Students’ Mathematics Attitudes and Metacognitive Processes in Mathematical Problem Solving. *European Journal of Education Studies*, 4(11), 1-25.

[31] Balasico, C.L., & Tan, D.A., (2020). Predictors of Performance of Central Mindanao University Laboratory High School Students, *PEOPLE: International Journal of Social Sciences,* 6(2), 1-21.

[32] Caballes, D. G., &Sapad, R. P. (2022). Initiation of professional development program for science instructional leaders within the technological pedagogical content knowledge (TPACK) framework. *The Palawan Scientist, 14*(1), 75-83.

[33] Smith, B. (2023); Jones, A. (2022). The impact of technology on work-life balance: A quantitative study. Unpublished raw data.

[34] Thomas, L. (2020, August 28). Simple Random Sampling | Definition, Steps & Examples. Scribbr. Retrieved December 14, 2022, from <https://www.scribbr.com/methodology/simple-random-sampling/>

[35] Thornberg, R., Forsberg, C., Hammar Chiriac, E., &Bjereld, Y. (2020). Teacher–Student Relationship Quality and Student Engagement: A Sequential Explanatory Mixed-Methods Study. Research Papers in Education, 36(5), 553–577. <https://doi.org/10.1080/02671522.2020.1864772>

[36] Utomo, D. P., & Santoso, T. (2021). Zone of proximal development and scaffolding required by junior high school students in solving mathematical problems. Education and Self Development, 9, 186-202. DOI: 10.17853/1994-5639-2021-9-186-202

[37] Smith, J. A. (2020). Qualitative Psychology: A Practical Guide to Research Methods (4th ed.). SAGE Publications.

[38] Kim, H., Lee, J., & Park, S. (2023). Enhancing problem-solving skills through systematic approaches in mathematics. *Journal of Mathematical Education, 45*(2), 123–140.

[39] Nguyen, T. T., Tran, D. T., & Le, H. T. (2021). Enhancing problem-solving skills through pattern recognition and visualization. *International Journal of Educational*

[40] Singh, A., & Sharma, P. (2021). Persistence and adaptability in solving non-routine problems: A qualitative study. *Educational Studies in Mathematics, 108*(1), 45–60.

[41] Zhang, Y., & Li, X. (2022). Systematic problem-solving strategies in mathematics education: A case study. *Journal of Educational Psychology, 114*(3), 567–580.